Joint life annuity payable annually in advance

The present value of a joint life annuity of 1 *pa* payable annually in advance to (x) and (y) is $\ddot{a}_{\overline{K_{xy}+1|}}$.

The EPV of a joint life annuity of 1 *pa* payable annually in advance to (x) and (y) is given by:

$$\ddot{a}_{xy} = E\left(\ddot{a}_{\overline{K_{xy}+1|}}\right) = \sum_{k=0}^{\infty} \ddot{a}_{\overline{k+1|}} P\left(K_{xy} = k\right) = \sum_{k=0}^{\infty} v^k \, {}_k p_{xy} = \sum_{k=0}^{\infty} v^k \, {}_k p_x \, {}_k p_y$$

The final expression for $\ddot{a}_{xy}$ holds because ${}_k p_{xy} = {}_k p_x \, {}_k p_y$, as we saw in Section 8.2.

Example 8.9

A joint life annuity of 1 *pa* is payable annually in advance to life aged 90 and a life aged 91. The effective annual rate of interest is 5%. Mortality for each life is assumed to follow the life table given below:

Age, x	l_x	d_x
90	100	25
91	75	35
92	40	40
93	0	0

Calculate the expected present value of this annuity.

Solution

Using the final formula in the box above, the expected present value is:

$$\ddot{a}_{90:91} = 1 + v\, p_{90}\, p_{91} = 1 + v \frac{l_{91}}{l_{90}} \times \frac{l_{92}}{l_{91}}$$

There are only 2 non-zero terms in the summation since ${}_k p_{91} = 0$ for $k \ge 2$. So:

$$\ddot{a}_{90:91} = 1 + 1.05^{-1} \times \frac{75}{100} \times \frac{40}{75} = 1.380952$$ ♦♦

If the two lives considered are a male and a female, where the male's mortality is assumed to follow the PMA92C20 mortality table and the female's mortality is assumed to follow the PFA92C20 mortality table, and the annual effective interest rate is 4%, then we are able to look up the value of $\ddot{a}_{xy}$ in the *Tables*.

An extract of the table of $\ddot{a}_{xy}$ values follows shortly. We have not shown the full table here – there are some columns missing on the right-hand side.

This table is set up based on the convention that the first age given in the subscript of the annuity function, x, is the male's age, and the second age given in the subscript, y, is the female's age. We will use this convention whenever we consider policies involving two lives.

It is important to be able to read values from this table accurately. The rows of the table relate to the male's age, *x*. The columns of the table relate to the age difference, *d*, where this is calculated as the female age, *y*, minus the male age, *x*. This definition is given at the top of the table, as indicated by one of the arrows.

PMA92C20 and PFA92C20

$\ddot{a}_{xy}$ for male (x) and female (y)

Age difference d $(= y - x)$

x \ d	−20	−10	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5
50	18.746	18.493	18.192	18.110	18.019	17.918	17.808	17.688	17.556	17.413	17.258	17.090	16.909
51	18.467	18.206	17.894	17.809	17.715	17.612	17.498	17.374	17.238	17.091	16.931	16.758	16.572
52	18.179	17.908	17.586	17.499	17.402	17.295	17.178	17.050	16.910	16.758	16.594	16.416	16.225
53	17.881	17.601	17.269	17.178	17.078	16.968	16.848	16.716	16.572	16.415	16.246	16.064	15.867
54	17.573	17.283	16.941	16.847	16.744	16.631	16.507	16.371	16.223	16.062	15.888	15.701	15.499
55	17.255	16.955	16.602	16.506	[illegible]	6.284	16.156	16.016	15.864	15.699	15.521	15.328	15.121
56	16.926	16.617	16.253	16.155	6.046	15.926	15.795	15.651	15.495	15.326	15.143	14.945	14.733
57	16.587	16.269	15.894	15.793	15.681	15.558	15.423	15.276	15.116	14.942	14.755	14.553	14.337
58	16.238	15.910	15.525	15.421	15.306	15.180	15.041	14.891	14.727	14.549	14.357	14.151	13.932
59	15.879	15.541	15.146	15.039	14.921	14.791	14.650	14.495	14.327	14.145	13.950	13.742	13.520
60	15.509	15.161	14.756	14.646	14.526	14.393	14.248	14.090	13.918	13.734	13.536	13.325	13.101
61	15.129	14.772	14.356	14.244	14.121	13.985	13.837	13.675	13.501	13.314	13.114	12.901	12.675
62	14.740	14.374	13.949	13.834	13.708	13.569	13.418	13.254	13.078	12.888	12.686	12.472	12.245
63	14.343	13.968	13.533	13.416	13.287	13.145	12.992	12.826	12.648	12.458	12.255	12.039	11.812
64	13.939	13.555	13.111	12.991	12.859	12.716	12.561	12.394	12.215	12.023	11.819	11.604	11.376
65	13.529	13.136	12.682	12.560	12.427	12.282	12.126	11.958	11.778	11.586	11.382	11.167	10.940
66	13.112	12.711	12.248	12.125	11.991	11.845	11.688	11.520	11.339	11.147	10.944	10.729	10.504
67	12.692	12.282	11.811	11.687	11.552	11.406	11.248	11.080	10.900	10.708	10.506	10.293	10.070
68	12.267	11.849	11.372	11.247	11.112	10.966	10.808	10.640	10.460	10.270	10.070	9.859	9.639
69	11.840	11.414	10.933	10.807	10.672	10.526	10.369	10.201	10.023	9.835	9.637	9.429	9.213
70	11.412	10.978	10.494	10.368	10.233	10.088	9.932	9.766	9.590	9.404	9.209	[illegible]	8.792
75	9.295	8.833	8.357	8.238	8.110	7.975	7.831	7.679	7.520	7.355	7.182	7.005	6.822
80	7.335	6.876	6.441	6.336	6.224	6.107	5.985	5.857	5.725	5.588	5.449	5.306	5.161
85	5.660	5.235	4.864	4.777	4.687	4.593	4.496	4.396	4.294	4.189	4.084	3.977	3.870
90	4.339	3.963	3.664	3.597	3.528	3.456	3.384	3.310	3.235	3.160	3.084	3.008	2.933
95	3.361	3.039	2.808	2.757	2.706	2.654	2.602	2.549	2.496	2.444	2.391	2.339	2.288
100	2.670	2.400	2.223	2.186	2.149	2.112	2.075	2.038	2.001	1.965	1.930	1.895	1.861

Example 8.10

Assuming that the male's mortality follows PMA92C20, the female's mortality follows PFA92C20, and that the annual effective interest rate is 4%, evaluate the following:

(i) $\ddot{a}_{55:51}$

(ii) $\ddot{a}_{70:73}$

Solution

(i) Here, the male age $x = 55$, and the age difference $d = 51 - 55 = -4$, so the value of the joint life annuity (indicated by one of the arrows) is:

$$\ddot{a}_{55:51} = 16.506$$

(ii) In this case, the male age $x = 70$, and the age difference $d = 73 - 70 = 3$, so the value of the joint life annuity (indicated by one of the arrows) is:

$$\ddot{a}_{70:73} = 9.209$$

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